MATLAB Course
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- MATLAB: **Mat**rix **Lab**oratory
- The Math Work Inc.
- See “Getting Started with MATLAB, version 6”

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Chapter 1: Elementary operations

- >> place for action in Matlab command window

- **Create matrix** $\mathbf{A} = \begin{pmatrix} 1 & 2 & 5 & 2 \\ 7 & 2 & 4 & 5 \\ 8 & 1 & 3 & 2 \end{pmatrix}$.

  >>A=[1 2 5 2;7 2 4 5;8 1 3 2]
- Show element **A**(1,3)

  >>A(1,3)
- Compute sum of columns

  >> sum (A)
- Operator colon ":",

  >>A(1,:)
- Use the colon for selection of parts of matrix A

  >>A(1,2:3)

- Building blocks of expressions
  - Variables

    >>x=[1 -2 3], x is a (1×3) array.
  - Numbers, real, integer, complex

    >>x=[1:1:8]
  - Operators
  - Functions
- Give examples

- Operators on two matrices **A** and **B** (if sizes correct, and symmetric if necessary)
  - **+** sum, >>A+B
  - **–** difference, >>A-B
  - ***** product, >>A*B
  - **/** division, >>A/B is equal to $\mathbf{AB}^{-1}$
  - **** division, >>A\B is equal to $\mathbf{A}^{-1}\mathbf{B}$
  - **^** power >>A^3 is equal to **A×A×A**
  - **‘** transpose, >>A’
  - **.^** element wise power, >>A.^2
  - **.*** element wise product, >>A.*B

- Functions:
  - abs absolute value
  - sqrt square route
  - exp exponent
  - sin sinus
  - size dimensions of a matrix
  - inv inverse of a matrix
  - det determinant of a matrix

Information about functions

Type:

For list of elementary mathematical functions

help elfun

For list of advanced mathematical and matrix functions

help specfun
help elmat
help matfun

For special function type:

help *function name*

For instance

>> **help zeros**

```
   ZEROS  Zeros array.
   ZEROS(N) is an N-by-N matrix of zeros.
   ZEROS(M,N) or ZEROS([M,N]) is an M-by-N matrix of zeros.
   ZEROS(M,N,P,...) or ZEROS([M N P ...]) is an M-by-N-by-P-
      by-...  array of zeros.
   ZEROS(SIZE(A)) is the same size as A and all zeros.

   See also EYE, ONES.
```

If you do not know the exact name of a function type:

lookfor ….

>> **help elfun**

Elementary math functions.

Trigonometric.

sin	- Sine.
sinh	- Hyperbolic sine.
asin	- Inverse sine.
asinh	- Inverse hyperbolic sine.
cos	- Cosine.
cosh	- Hyperbolic cosine.
acos	- Inverse cosine.
acosh	- Inverse hyperbolic cosine.
tan	- Tangent.
tanh	- Hyperbolic tangent.
atan	- Inverse tangent.
Atan2	- Four quadrant inverse tangent.
atanh	- Inverse hyperbolic tangent.
sec	- Secant.
sech	- Hyperbolic secant.
asec	- Inverse secant.
asech	- Inverse hyperbolic secant.
csc	- Cosecant.
csch	- Hyperbolic cosecant.
acsc	- Inverse cosecant.
acsch	- Inverse hyperbolic cosecant.
cot	- Cotangent.
coth	- Hyperbolic cotangent.
acot	- Inverse cotangent.
acoth	- Inverse hyperbolic cotangent.

Exponential.

exp	- Exponential.
log	- Natural logarithm.
log10	- Common (base 10) logarithm.
log2	- Base 2 logarithm and dissect floating point number.
pow2	- Base 2 power and scale floating point number.
realpow	- Power that will error out on complex result.
reallog	- Natural logarithm of real number.
realsqrt	- Square root of number greater than or equal to zero.
sqrt	- Square root.
nextpow2	- Next higher power of 2.

Complex.

abs	- Absolute value.
angle	- Phase angle.
complex	- Construct complex data from real and imaginary parts.
conj	- Complex conjugate.

imag - Complex imaginary part.
real - Complex real part.
unwrap - Unwrap phase angle.
isreal - True for real array.
cplxpair - Sort numbers into complex conjugate pairs.

Rounding and remainder.
fix - Round towards zero.
floor - Round towards minus infinity.
ceil - Round towards plus infinity.
round - Round towards nearest integer.
mod - Modulus (signed remainder after division).
rem - Remainder after division.
sign - Signum.

>> **help specfun**

Specialized math functions.

Specialized math functions
airy - Airy functions.
besselj - Bessel function of the first kind.
bessely - Bessel function of the second kind.
besselh - Bessel functions of the third kind (Hankel function).
besseli - Modified Bessel function of the first kind.
besselk - Modified Bessel function of the second kind.
beta - Beta function.
betainc - Incomplete beta function.
betaln - Logarithm of beta function.
ellipj - Jacobi elliptic functions.
ellipke - Complete elliptic integral.
erf - Error function.
erfc - Complementary error function.
erfcx - Scaled complementary error function.
erfinv - Inverse error function.
expint - Exponential integral function.
gamma - Gamma function.
gammainc - Incomplete gamma function.
gammaln - Logarithm of gamma function.
psi - Psi (polygamma) function.
legendre - Associated Legendre function.
cross - Vector cross product.
dot - Vector dot product.

Number theoretic functions.

factor	- Prime factors.
Isprime	- True for prime numbers.
primes	- Generate list of prime numbers.
gcd	- Greatest common divisor.
lcm	- Least common multiple.
rat	- Rational approximation.
rats	- Rational output.
perms	- All possible permutations.
nchoosek	- All combinations of N elements taken K at a time.
factorial	- Factorial function.

Coordinate transforms.

cart2sph	- Transform Cartesian to spherical coordinates.
cart2pol	- Transform Cartesian to polar coordinates.
pol2cart	- Transform polar to Cartesian coordinates.
sph2cart	- Transform spherical to Cartesian coordinates.
hsv2rgb	- Convert hue-saturation-value colors to red-green-blue.
rgb2hsv	- Convert red-green-blue colors to hue-saturation-value.

>> **help elmat**

Elementary matrices and matrix manipulation.

Elementary matrices.

zeros	- Zeros array.
ones	- Ones array.
eye	- Identity matrix.
repmat	- Replicate and tile array.
rand	- Uniformly distributed random numbers.
randn	- Normally distributed random numbers.
linspace	- Linearly spaced vector.
logspace	- Logarithmically spaced vector.
freqspace	- Frequency spacing for frequency response.
meshgrid	- X and Y arrays for 3-D plots.
:	- Regularly spaced vector and index into matrix.

Basic array information.

size	- Size of array.
length	- Length of vector.
ndims	- Number of dimensions.
numel	- Number of elements.
disp	- Display matrix or text.
isempty	- True for empty array.
isequal	- True if arrays are numerically equal.
isequalwithequalnans	- True if arrays are numerically equal.
isnumeric	- True for numeric arrays.

islogical - True for logical array.
logical - Convert numeric values to logical.

Matrix manipulation.
cat - Concatenate arrays.
reshape - Change size.
diag - Diagonal matrices and diagonals of matrix.
blkdiag - Block diagonal concatenation.
tril - Extract lower triangular part.
triu - Extract upper triangular part.
fliplr - Flip matrix in left/right direction.
flipud - Flip matrix in up/down direction.
flipdim - Flip matrix along specified dimension.
rot90 - Rotate matrix 90 degrees.
: - Regularly spaced vector and index into matrix.
find - Find indices of nonzero elements.
end - Last index.
sub2ind - Linear index from multiple subscripts.
ind2sub - Multiple subscripts from linear index.

Multi-dimensional array functions.
ndgrid - Generate arrays for N-D functions and interpolation.
permute - Permute array dimensions.
ipermute - Inverse permute array dimensions.
shiftdim - Shift dimensions.
circshift - Shift array circularly.
squeeze - Remove singleton dimensions.

Special variables and constants.
ans - Most recent answer.
eps - Floating point relative accuracy.
realmax - Largest positive floating point number.
realmin - Smallest positive floating point number.
pi - 3.1415926535897....
i, j - Imaginary unit.
inf - Infinity.
NaN - Not-a-Number.
isnan - True for Not-a-Number.
isinf - True for infinite elements.
isfinite - True for finite elements.
why - Succinct answer.

Specialized matrices.
compan - Companion matrix.
gallery - Higham test matrices.
hadamard - Hadamard matrix.
hankel - Hankel matrix.
hilb - Hilbert matrix.

```
  invhilb     - Inverse Hilbert matrix.
  magic       - Magic square.
  pascal      - Pascal matrix.
  rosser      - Classic symmetric eigenvalue test problem.
  toeplitz    - Toeplitz matrix.
  vander      - Vandermonde matrix.
  wilkinson   - Wilkinson's eigenvalue test matrix.
```

>> **help matfun**

Matrix functions - numerical linear algebra.

Matrix analysis.

```
  norm        - Matrix or vector norm.
  normest     - Estimate the matrix 2-norm.
  rank        - Matrix rank.
  det         - Determinant.
  trace       - Sum of diagonal elements.
  null        - Null space.
  orth        - Orthogonalization.
  rref        - Reduced row echelon form.
  subspace    - Angle between two subspaces.
```

Linear equations.

```
  \ and /     - Linear equation solution; use "help slash".
  inv         - Matrix inverse.
  rcond       - LAPACK reciprocal condition estimator
  cond        - Condition number with respect to inversion.
  condest     - 1-norm condition number estimate.
  normest1    - 1-norm estimate.
  chol        - Cholesky factorization.
  cholinc     - Incomplete Cholesky factorization.
  lu          - LU factorization.
  luinc       - Incomplete LU factorization.
  qr          - Orthogonal-triangular decomposition.
  lsqnonneg   - Linear least squares with nonnegativity constraints.
  pinv        - Pseudoinverse.
  lscov       - Least squares with known covariance.
```

Eigenvalues and singular values.

```
  eig         - Eigenvalues and eigenvectors.
  svd         - Singular value decomposition.
  gsvd        - Generalized singular value decomposition.
  eigs        - A few eigenvalues.
  svds        - A few singular values.
  poly        - Characteristic polynomial.
  polyeig     - Polynomial eigenvalue problem.
  condeig     - Condition number with respect to eigenvalues.
```

hess - Hessenberg form.
qz - QZ factorization for generalized eigenvalues.
schur - Schur decomposition.

Matrix functions.
expm - Matrix exponential.
logm - Matrix logarithm.
sqrtm - Matrix square root.
funm - Evaluate general matrix function.

Factorization utilities
qrdelete - Delete a column or row from QR factorization.
qrinsert - Insert a column or row into QR factorization.
rsf2csf - Real block diagonal form to complex diagonal form.
cdf2rdf - Complex diagonal form to real block diagonal form.
balance - Diagonal scaling to improve eigenvalue accuracy.
planerot - Givens plane rotation.
Cholupdate - rank 1 update to Cholesky factorization.
qrupdate - rank 1 update to QR factorization.

Working with matrices

Basic matrices

- zeros
- ones
- rand random numbers
- randn random numbers from standard normal distribution

Basic functions of a matrix

- mean
- corrcoef
- cov
- var
- diag diag(**A**) is a vector with diagonal elements of **A**
- diag(diag) diag(diag(**A**)) is a diagonal matrix with elements diag(**A**)

for description type: help *function name*

Search in MATLAB

Help
MATLAB help
index
eg random number generators
functions
normrnd - normalized random numbers
unidrnd - discrete uniform random numbers
unifrnd - continuous uniform random numbers

eg randn - random numbers normally standardized

Exercises Chapter 1

1: Generate two matrices **A** and **B** of order (5 x 5), in which matrix **A** is a symmetric matrix, and

1a: compute **A** + **B**, **A** - **B**, **AB**.
1b: compute the determinant of **A**.
1c: compute the inverse of **A** and verify that this inverse is correct.
1d: compute the eigenvalues and eigenvectors of matrix **A**.
1e: Generate a matrix **C** of order (6 x 3) and compute the singular value decomposition (svd) of **C**. Verify that this decomposition is correct by comparing the results with the eigenvectors and eigenvalues of **C′C** and **CC′.**

2: The scores of 5 subjects are given in matrix **X** for the independent variables and for the dependent variable in vector **y**.:

$$\mathbf{y} = \begin{pmatrix} 10 \\ 20 \\ 17 \\ 12 \\ 11 \end{pmatrix} \text{ and } \mathbf{X} = \begin{pmatrix} 1 & 6 & 28 \\ 1 & 12 & 40 \\ 1 & 10 & 32 \\ 1 & 8 & 36 \\ 1 & 9 & 34 \end{pmatrix}.$$

2a: Estimate the regression weights in a multiple regression with $\mathbf{y} = \mathbf{Xb} + \mathbf{e}$. Here it holds: $\hat{\mathbf{b}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$.
2b: Compute the vector of errors and the variance of the errors.
2c: Compute the eigenvectors V and the eigenvalues **D** of **X′X**. Next compute $(\mathbf{VDV'})^{-1}$ and compare this with $(\mathbf{X'X})^{-1}$.
2d: Compute $(\mathbf{X'X})^2$ and $\mathbf{VD^2V'}$ and compare them.
.
3: The scores of 4 subjects on two variables are given in the following matrix.

$$\mathbf{X} = \begin{pmatrix} 2 & 5 \\ 5 & 5 \\ 2 & 4 \\ 3 & 2 \end{pmatrix}.$$

3a: Transform this matrix into a matrix in which the scores are in deviations of the column means.
3b: Compute the covariance matrix (devide by *n*) and apply the MATLAB function cov to **X**. Can you explain the difference in results?
3c: Compute the standardized **X**-scores (means 0 and variances 1) and place them in **Z**.
3d: Compute the correlation matrix of the **X**-scores by applying the MATLAB function corrcoef to **X** and compare it with the covariance matrix of **Z** (devided by *n*).
3e: Carry out a principal component analysis on this correlation matrix and show the eigenvectors **V**, the eigenvalues **D** and the component scores **ZV**. Are the component scores standardized?
3f: Compute the component loadings, which are the correlations between **Z** and the component scores. Compare the component loadings with $\mathbf{V'D}^{½}$. Why are they similar?
3g: Apply the function corrcoef to [**Z,Z*V**]. Do you find the component loadings? What does the other blocks mean?